Downlink Beamforming for Other-cell Interference Mitigation in Correlated MISO Channels

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***Abstract* - In this paper, we consider a multi-antenna technique that can mitigate other cell interference (OCI) in correlated multiple-input single-output (MISO) cellular environments. By exploiting the spatial channel information with adjacent cells, the proposed scheme generates a beamforming weight that yields less interference to other users in the adjacent cells, while increasing the signal power to the target user. To reduce the implementation complexity, we consider the sharing of channel information with a minimum number of adjacent cells. Finally, the performance of the proposed scheme is verified by computer simulation. The simulation results show that the proposed scheme provides noticeable performance improvement particularly in high spatial correlation environments.**

I. INTRODUCTION

The use of multiple antennas at both sides of the link, so called multi-input multi-output (MIMO), recently has drawn considerable attention for future wireless communications. When the channel gains between the transmit and receive antennas are independent and identically distributed (i.i.d.), the channel capacity increases in linear proportion to the minimum number of transmit and receive antennas, even though the transmitter has no information on the channel [1-3]. In practice, however, the channel gains are often spatially correlated to each other [4]. Previous studies show that the channel capacity significantly deteriorates in the presence of spatial correlation when no channel information is available [5,6].

When the transmitter acquires long-term characteristics of the channel, it can improve the performance by making the use of spatial correlation, so-called eigen-beamforming [7,8]. In spatially correlated channel environments, the channel energy is mostly concentrated on a small number of eigen-dimensions of the channel covariance matrix. Conventional eigen-beamforming schemes transmit the data in the direction of the eigenvectors of the channel covariance matrix with power allocation [7,8]. These transmission schemes can maximally deliver the signal power to the target user in an average sense [9,10]. Since relative geometry of propagation paths is well reflected in the channel covariance matrix, the transmitter can achieve a noticeable performance gain compared to the open loop system by using spatial correlation information of the channel. Moreover, transmitter can reliably achieve the this long-term channel information through a low rate feedback channel, requiring a marginal increase of the feedback signaling overhead.

In cell boundary environments, the capacity of a cellular system is mainly limited by the interference signal from other cells, so-called other-cell interference (OCI). Even when the user can receive the signal with a good quality from the serving cell, it may not properly communicate with its serving cell due to large interference from adjacent cells. Thus, it is highly required not only to maximize the desired signal power but also to minimize the interference from adjacent cells.

In this paper, we propose a new beamforming scheme for the improvement of the signal-to-interference plus noise power ratio (SINR) near the cell boundary environments. The proposed scheme considers the sharing of spatial correlation information with adjacent cells for the generation of beam weight that yields reduced interference to other users in the adjacent cells, while maximally delivering the signal power to the target user. In order to reduce the implementation complexity, we also consider the sharing of spatial correlation information with a minimum number of adjacent cells.

The rest of the paper is organized as follows. Section II represents the system model and Section III describes the proposed scheme. Section IV verifies the performance of the proposed scheme by computer simulation. Finally, Section V summarizes conclusions.

II. SYSTEM MODEL

Consider the downlink of a multi-user wireless system where base stations (BSs) are synchronized to each other with universal frequency reuse. We assume that each BS transmits the signal using M transmit antennas with beamforming and each user receives the signal using a single receive antenna. We also assume that the user can estimate the channel information using a common pilot signal and the BS can get the channel information from the user through a feedback signaling channel. To reduce the amount of feedback signaling burden, we consider the use of averaged channel characteristics such as the spatial correlation and the average received signal power.

Assuming a cellular network comprising N_c cells, the instantaneous SINR $\gamma_{i,k}$ of user k in cell i can be represented as

$$\gamma_{i,k} = \frac{\left|\mathbf{h}_{i,k}\mathbf{w}_i\right|^2 P_{i,k}}{\sum_{\mathrm{j=1,j\neq i}}^{N_c} \left|\mathbf{h}_{j,k}\mathbf{w}_j\right|^2 P_{j,k} + N_0} \quad (1)$$

where $\mathbf{h}_{i,k}$ denotes the $(1\times M)$ channel vector from cell i to user k whose elements can be expressed by zero mean complex Gaussian random variables with unit variance, $\mathbf{w}_i$ denotes the $(M\times 1)$ beamforming weight vector of cell i, and $P_{i,k}$ and N_0 respectively denote the average signal power from cell i to user k and the average power of additive white Gaussian noise (AWGN). The Frobenius norm of $\|\mathbf{w}_i\|_F$ for $i=1,...,N_c$ is set to one to preserve the transmission power.

For ease of description, define an active set Ω_k of user k by a set of cells that strongly affect the signal reception of user k. The active set comprises a primary cell and non-primary cells. The primary cell in the Ω_k is a cell actively serving user k and the non-primary cells in the Ω_k are cells affecting user k as strong interference sources. The performance of users near the cell boundary can be improved by mitigating the OCI. It can be seen from (1) that the OCI heavily depends on the beamforming weight of other cells and most of them may come from the non-primary cells. Thus, performance of users near the cell boundary can be improved by actively handling the beamforming weight of the non-primary cells as well as that of the primary cell.

In a spatially correlated Rayleigh fading environment, $\mathbf{h}_{i,k}$ can be expressed as [11]

$$\mathbf{h}_{i,k} = \tilde{\mathbf{h}}_{i,k}\mathbf{R}_{i,k}^{1/2} \tag{2}$$

where $\tilde{\mathbf{h}}_{i,k}$ denotes the uncorrelated virtual channel vector from cell i to user k whose elements are expressed by i.i.d. zero-mean complex Gaussian random variables with unit variance, and $\mathbf{R}_{i,k}$ denotes the $(M\times M)$ channel covariance matrix defined by

$$\mathbf{R}_{i,k} = E\left\{\mathbf{h}_{i,k}^{*}\mathbf{h}_{i,k}\right\}. \tag{3}$$

Here, the superscript * denotes conjugate transpose. Since $\mathbf{R}_{i,k}$ is a Hermitian matrix, it can be represented as

$$\mathbf{R}_{i,k} = \mathbf{V}_{i,k}\mathbf{\Sigma}_{i,k}\mathbf{V}_{i,k}^{*} \tag{4}$$

where the diagonal elements of diagonal matrix $\mathbf{\Sigma}_{i,k}$ and the columns of unitary matrix $\mathbf{V}_{i,k}$ are the eigenvalues and the corresponding eigenvectors of $\mathbf{R}_{i,k}$, respectively. We define $\mathbf{w}_{i,k}^{\max}$ and $\mathbf{w}_{i,k}^{\min}$ by the eigenvector corresponding to the maximum eigenvalue $\lambda_{i,k}^{\max}$ and the minimum eigenvalue $\lambda_{i,k}^{\min}$ of $\mathbf{R}_{i,k}$, respectively. Obviously, $\mathbf{w}_{i,k}^{\max}$ is the conventional eigen-beamforming [1] weight maximizing the average SNR, while $\mathbf{w}_{i,k}^{\min}$ is the beamforming weight minimizing the average SNR.

[1] It can be shown that the eigen-beamforming in the direction corresponding to the largest eigenvalue of the channel covariance matrix is asymptotically optimum as the SNR decreases to zero [9]. Since we are concerned on users near the cell boundary, we consider the use of eigen-beamforming that allocates all power to the strongest eigen-dimention of channel.

III. PROPOSED BEAMFORMING SCHEME

In the proposed scheme, we assume that each BS generates a beamforming weight at each time slot and users report their SINR for the given beamforming weight. This procedure is similar to that in [12], but the proposed scheme generates the beamforming weight in coordination with adjacent BSs to mitigate the OCI, not in a random manner as in [12]. Overall performance can be maximized by generating the beamforming weight in response to instantaneous channel condition. However, this may cause heavy feedback signaling overhead in the uplink. To alleviate this problem, we consider the generation of beamforming weight based on the average channel characteristics, (i.e., the channel covariance information). By sharing the user's channel covariance information with the adjacent BSs, each BS can generate the beamforming weight in a coordinated manner. However, the instantaneous channel condition of users may differ from the average one. Thus, even though each BS generates the beamforming weight in favor of its target user based on the average characteristics, it may allocate its resource to a user experiencing the best instantaneous SINR at each time slot. As a consequence, the proposed scheme can obtain a multi-user diversity gain as the number of users increases.

To mitigate the OCI effect to a user, consider the case that the non-primary cells generate beams corresponding to the minimum eigenvalue of the channel covariance matrix, while making the primary cell generates a beam corresponding to the maximum eigenvalue of channel covariance matrix. This case is only meaningful when all the BSs generate their beams considering only this target user. In practice, however, each BS needs to generate its beamforming weight for its own target user, yielding conflicting issues among them.

To resolve this problem, we introduce the concept of *jamming* as in [13]. Let define the average jamming power J_i from cell i by the total average interference power experienced by target users of all other cells. Then, J_i can be represented as

$$\begin{aligned} J_i &= \sum_j E\left\{\mathbf{w}_i^{*}\mathbf{h}_{i,j}^{*}\mathbf{h}_{i,j}\mathbf{w}_i\right\}P_{i,j} \\ &= \sum_j \mathbf{w}_i^{*}\mathbf{R}_{i,j}\mathbf{w}_i P_{i,j}\,. \end{aligned} \tag{5}$$

Thus, cell i may need to adjust $\mathbf{w}_i$ considering the interference power experienced by all target users of other cells. In practice, however, it may be sufficient to only consider users whose active set includes cell i as a non-primary cell because most of J_i will be delivered to these users. Then, the optimum beamforming weight $\mathbf{w}_{i,k}^{opt}$ of cell i for target user k can be determined based on a maximum average signal-to-jamming and noise power ratio (SJNR) criterion as [13]

$$\mathbf{w}_{i,k}^{opt} = \arg \max_{\mathbf{w}_i : \|\mathbf{w}_i\|_F = 1} \frac{\mathbf{w}_i^* \mathbf{R}_{i,k} \mathbf{w}_i P_{i,k}}{\sum_{j \in \Psi_i} \mathbf{w}_i^* \mathbf{R}_{i,j} \mathbf{w}_i P_{i,j} + I_k + N_0}$$
$$= \arg \max_{\mathbf{w}_i : \|\mathbf{w}_i\|_F = 1} \frac{\mathbf{w}_i^* \mathbf{R}_{i,k} P_{i,k} \mathbf{w}_i}{\mathbf{w}_i^* \left(\sum_{j \in \Psi_i} \mathbf{R}_{i,j} P_{i,j} \right) \mathbf{w}_i + I_k + N_0} \quad (6)$$

where Ψ_i denotes the set of target users of other cells whose active set includes cell i as a non-primary cell, and I_k denotes the average interference power from neighbor cells not belonging to Ω_k. Since (6) is a generalized Rayleigh quotient problem, the optimum beamforming weight $\mathbf{w}_{i,k}^{opt}$ can be obtained by [14]

$$\mathbf{w}_{i,k}^{opt} = \mathbf{V}_{\max} \left(\left(\sum_{j \in \Psi_i} \mathbf{R}_{i,j} P_{i,j} + \left(I_k + N_0 \right) \mathbf{I}_M \right)^{-1} \mathbf{R}_{i,k} P_{i,k} \right) \quad (7)$$

where $\mathbf{V}_{\max}\left(\mathbf{X}\right)$ denotes the eigenvector corresponding to the largest eigenvalue of $\mathbf{X}$ and $\mathbf{I}_M$ denotes an $\left(M \times M\right)$ identity matrix. When cell i generates a beam for its target user k, there can be special cases summarized as follows.

1) When all users in Ψ_i have an identity channel covariance matrix corresponding to cell i (i.e., no spatial channel correlation between cell i and users in Ψ_i), it can be seen that $\sum_{j \in \Psi_i} \mathbf{R}_{i,j} P_{i,j} = \sum_{j \in \Psi_i} \mathbf{I}_M P_{i,j}$ and $\mathbf{w}_{i,k}^{opt}$ is simply equal to conventional eigen-beamforming weight $\mathbf{w}_{i,k}^{\max}$. Since all users in Ψ_i may experience the same interference regardless of $\mathbf{w}_{i,k}^{opt}$ in an average sense, cell i simply maximizes the average signal power of its target user k.
2) If $\mathbf{R}_{i,k} = \mathbf{I}_M$, then $\mathbf{w}_{i,k}^{opt} = \mathbf{V}_{\max} \left(\left(\sum_{j \in \Psi_i} \mathbf{R}_{i,j} P_{i,j} \right)^{-1} P_{i,k} \right)$. This means that the target user k of cell i has no preference on the beamforming weight in an average sense. Thus, cell i simply needs to generate a beam to minimize the interference to users in Ψ_i. In addition, only a single user j in Ψ_i does not have an identity matrix as the channel covariance matrix corresponding to cell i, yielding $\mathbf{w}_{i,k}^{opt}$ to $\mathbf{w}_{i,j}^{\min}$.
3) When all the channels with cell i are spatially uncorrelated (i.e., $\mathbf{R}_{i,k} = \mathbf{I}_M$ for all k), it is optimum to use random beams.

In summary, the overall procedure of the proposed scheme can be described as

1. Each BS determines its target user and corresponding time slot according to its scheduling policy. In order to satisfy the QoS of users, for example, determines order of target users by round-robin manner.
2. Each BS exchanges order of target users with corresponding channel information (i.e., the product of the channel covariance matrix and the corresponding average signal power) with the adjacent BSs.

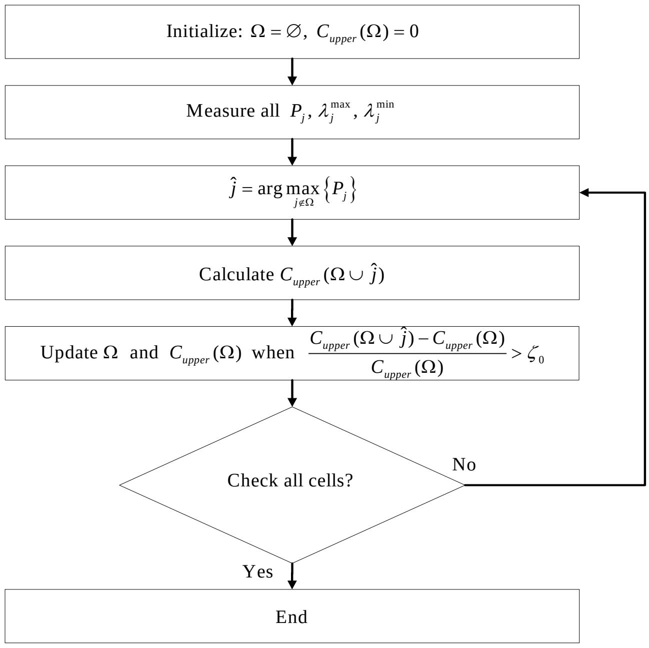


Fig. 1. Procedure of determining the active set.

3. Each BS generates a beam based on the maximum average SJNR criterion in (7) and gets the instantaneous SINR of the corresponding beam from all users in its cell.
4. Each BS selects the best user based on the reported instantaneous SINR and transmits the data using the corresponding beamforming weight. Then, iterate procedure 3 and 4.

As the number of non-primary cells in the active set increases, the interference can be better controlled at an expense of increased implementation complexity. However, since the proposed scheme only uses the spatial correlation information, it may not noticeably achieve the performance improvement by suppressing the interference from the non-primary cells with a small spatial correlation. Thus, it may be practical to only consider cells as the non-primary cells that provide large spatial correlation as well as strong signal power. Considering both the system complexity and the performance enhancement, we can determine and renew the active set as illustrated in Fig. 1, where $C_{upper}(\Omega)$ represents an upper bound of the performance in single user environments for a given active set Ω. Here, we omit the user index k for simplicity of notation.

1. Initialize the parameters with $\Omega = \varnothing$ and $C_{upper}(\Omega) = 0$, where $\varnothing$ denotes an empty set .
2. The user measures P_j, $\lambda_j^{\max}$ and $\lambda_j^{\min}$ from all cells $\{j\}$ in synchronization with itself.
3. Choose a cell $\hat{j}$ having the maximum average signal power, but not belonging to Ω.
4. Calculate the $C_{upper}(\Omega \cup \hat{j})$ when cell $\hat{j}$ is added to

TABLE I
SIMULATION PARAMETERS

Parameters	Setting
Cell configuration	19 cells (two-tier)
Cell radius	1 km
Antenna configuration	4×1
Path-loss exponent	4
Fading channel	Rayleigh fading
Link adaptation	Ideal (i.e., using the Shannon's capacity formula)
Occurrence of an outage stage	When the SINR is less than -5 dB

Ω. It can be calculated using (A.10) and (A.11) in Appendix.

5. Define the performance enhancement indicator by $\zeta \triangleq \left(C_{upper}(\Omega \cup \hat{j}) - C_{upper}(\Omega)\right) / C_{upper}(\Omega)$. If ζ is larger than a pre-determined threshold ζ_0, cell $\hat{j}$ is added to Ω.
6. If the user completely checks all the cells in synchronization, then stop. Else, go to step 3.

Then, the user reports the newly determined active set list with the corresponding product of the channel covariance matrix and the average signal power to the primary cell. Since the active set is determined based on long-term channel information, it can be renewed with a time period much longer than the time slot with low computational complexity. This long-term channel information can reliably be exchanged among the cells in the active set through a high-speed backbone network, requiring a marginal increase of the signaling burden.

IV. PERFORMANCE EVALUATION

We verify the performance of the proposed scheme in center cell with a two-tiered hexagonal shaped cellular network by computer simulation. Assuming the use of a (4×1) antenna configuration, we consider the signal transmission over a spatially correlated Rayleigh fading channel with simple channel covariance matrix $\mathbf{R}$ given by

$$\mathbf{R} = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho^* & 1 & \rho & \rho^2 \\ \rho^{2^*} & \rho^* & 1 & \rho \\ \rho^{3^*} & \rho^{2^*} & \rho^* & 1 \end{bmatrix} \quad (8)$$

where ρ represents the complex-valued spatial correlation coefficient between the adjacent antennas. We assume that users have the same number of N_Ω cells in their active set and experience the same correlation magnitude $|\rho|$ for all cells in their active set. The simulation environment is summarized in TABLE I.

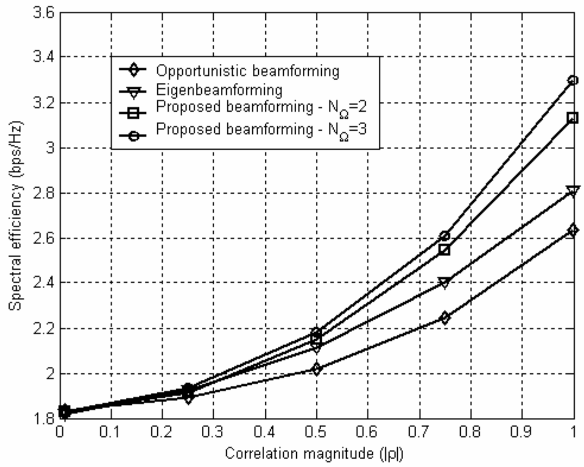


Fig. 2. Performance of the proposed scheme according to $|\rho|$.

To verify the validation of the proposed scheme, we compare the performance of the proposed scheme with two conventional beamforming schemes working with partial channel information; opportunistic beamforming and eigen-beamforming scheme. The beamforming weight of the eigen-beamforming is generated only considering the target user (i.e., corresponding to the special case of the proposed beamforming scheme with $N_\Omega = 1$).

Fig. 2 depicts the performance of the proposed scheme in terms of the correlation magnitude when 16 users are uniformly distributed near the cell boundary of the center cell. It can be seen that when the correlation magnitude is small (i.e. weekly correlated channel environment), the proposed scheme provides performance similar to the conventional beamforming schemes regardless of N_Ω. This is mainly because both the proposed and the eigen-beamforming scheme provide a marginal performance gain by exploiting the spatial correlation in weekly correlated channel environments. On the other hands, when the correlation magnitude is large, the proposed scheme and the eigen-beamforming can achieve a large performance gain by increasing the average power of the target signal compared to the opportunistic beamforming. Moreover, since the proposed scheme can better control the interference from the adjacent cells, it can outperform the eigen-beamforming scheme. The improvement becomes larger as N_Ω increases.

Fig. 3 depicts the multi-user diversity gain of the proposed scheme according to the number of users. It can be seen that when $|\rho| = 0.01$, the performance of the three schemes is almost the same because they generate the beam weight almost in a random manner. In this case, however, they can still achieve the multi-user diversity gain by allocating the resource to a user in the best channel condition. On the other hands, when $|\rho| = 0.99$, the proposed scheme generates the beamforming weight in favor of the target user and thus it can outperform the conventional schemes particularly when the number of users is small. Since the proposed scheme achieves a

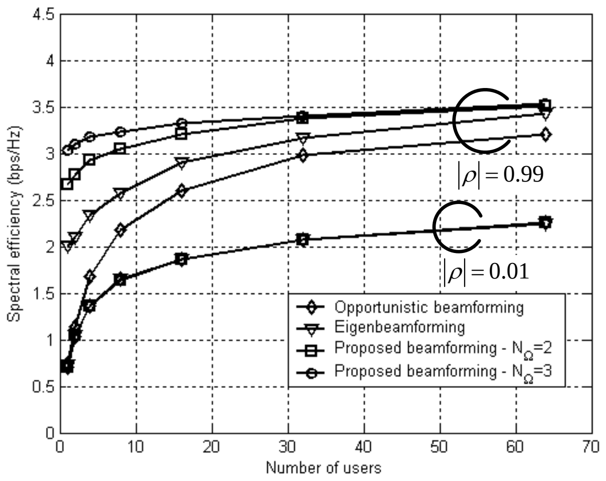
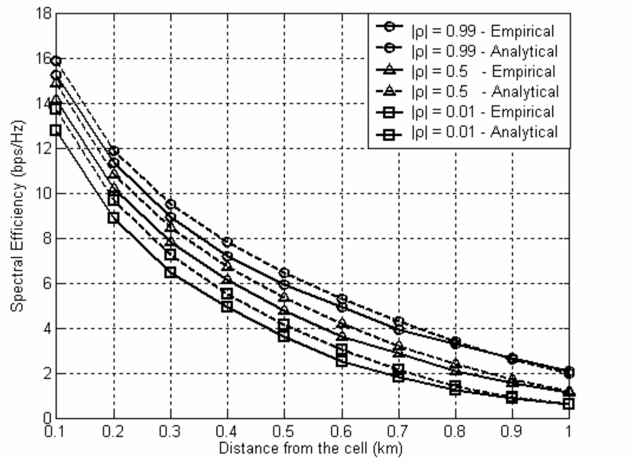
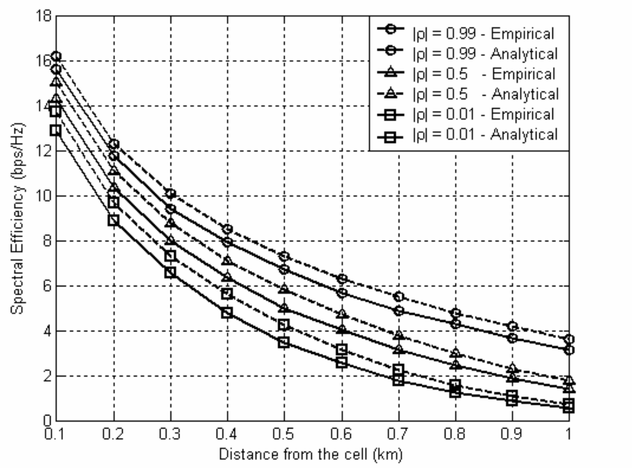


Fig. 3. Performance of the proposed scheme according to the number of users.



(a) When $N_{\Omega} = 2$



(b) When $N_{\Omega} = 3$

Fig. 4. Analytic upper bound of the spectral efficiency of the proposed scheme.

large SINR improvement from the beamforming by exploiting highly correlated channel characteristics, it is less affected by the multi-user diversity gain unlike in low channel correlation environments.

To verify the accuracy of the upper bound analysis in Appendix, Fig. 4 depicts the analytic upper bound (A.10) along with the simulation results. It can be seen that the analytic upper bound agrees well with the simulation results. However, when $N_{\Omega} = 2$, the analytic upper bound is slightly less than the simulation results near the cell boundary. In the analysis, the total interference power from cells not belonging to the active set is approximated by the average power I_k. Near the cell boundary region, the proposed scheme with $N_{\Omega} = 2$ may not sufficiently consider the most of significant interference terms, making the analysis somewhat inaccurate due to the effect of time variation of total interference. Nonetheless, since the difference between the simulation results and analytic upper bound is small, the analytic upper bound can effectively be applied to the generation of the active set.

V. CONCLUSION

We have proposed a new beamforming scheme that improves the SINR in cell boundary environments. By exploiting the spatial correlation information of users, the proposed scheme generates less interference to users in the adjacent cells, while maximizing the transmit signal power to the target user. To reduce the system complexity, the proposed scheme considers the sharing of the channel information with adjacent cells by minimizing the number of cells in the active set using an analytic performance bound. The simulation results show that the proposed scheme provides noticeable performance improvement over the conventional beamforming schemes particularly in high spatial correlation environments.

APPENDIX

AN UPPER BOUND OF PERFORMANCE IN SINGLE USER ENVIRONMENTS

Consider the special case where the primary cell of user k generates a beamforming weight that maximizes the average power of the target signal and the non-primary cells generate beamforming weights that minimize the average interference power to user k. Then, the instantaneous SINR $\gamma_{i,k}$ of user k signal received from the primary cell i can be represented as

$$\gamma_{i,k} = \frac{\left|\mathbf{h}_{i,k}\mathbf{w}_{i,k}^{\max}\right|^2 P_{i,k}}{\sum_{j\in\Omega_k,\, j\neq i}^{N_c}\left|\mathbf{h}_{j,k}\mathbf{w}_{j,k}^{\min}\right|^2 P_{j,k} + \sum_{j\notin\Omega_k}^{N_c}\left|\mathbf{h}_{j,k}\right|^2 P_{j,k} + N_0} \qquad \text{(A.1)}$$

where Ω_k denotes the active set of user k. Since the second term in the denominator of (A.1) is a sum of large number of independent signals with similar average power, it is less affected by the time fluctuation and can be approximated by its average power I_k.

Letting $I'_k \triangleq I_k + N_o$, (A.1) can further be approximated as

$$\gamma_{i,k} \approx \frac{\left|\mathbf{h}_{i,k}\mathbf{w}_{i,k}^{\max}\right|^2 P_{i,k}}{\sum_{j\in\Omega_k,\, j\neq i}^{N_c}\left|\mathbf{h}_{j,k}\mathbf{w}_{j,k}^{\min}\right|^2 P_{j,k} + I'_k} \triangleq \frac{X}{Y' + I'_k}. \tag{A.2}$$

$\tilde{\mathbf{h}}_{i,k}$ in (2) can be decomposed using orthonormal bases $\left\{\mathbf{O}_{k1}, \mathbf{O}_{k2}, \cdots, \mathbf{O}_{kM} \in \mathbb{C}^{1\times M}\right\}$ as

$$\tilde{\mathbf{h}}_{i,k} = \alpha_{k1}\mathbf{O}_{k1} + \alpha_{k2}\mathbf{O}_{k2} + \cdots + \alpha_{kM}\mathbf{O}_{kM}. \tag{A.3}$$

Since the elements of $\tilde{\mathbf{h}}_{i,k}$ can be represented by i.i.d. zero mean complex Gaussian random variables with unit variance, $\left\{\alpha_{k1}, \cdots, \alpha_{kM}\right\}$ can also be modeled by i.i.d. zero mean complex Gaussian random variables with unit variance for any orthonormal bases.

Letting the first orthonormal basis be $\mathbf{w}_{i,k}^{\max}$ (i.e., $\mathbf{O}_{k1} = \mathbf{w}_{i,k}^{\max}$), it can be shown that

$$\begin{aligned} \mathbf{h}_{i,k}\mathbf{w}_{i,k}^{\max}\sqrt{P_{i,k}} &= \tilde{\mathbf{h}}_{i,k}\left(\mathbf{R}_{i,k}\right)^{1/2}\mathbf{w}_{i,k}^{\max}\sqrt{P_{i,k}} \\ &= \sqrt{\lambda_{i,k}^{\max} P_{i,k}}\,\alpha_{k1}. \end{aligned} \tag{A.4}$$

Since $\mathbf{h}_{i,k}\mathbf{w}_{i,k}^{\max}\sqrt{P_{i,k}}$ is a zero-mean complex Gaussian random variable with variance $\lambda_{i,k}^{\max} P_{i,k}$, X can be modeled as a chi-square random variable with two degrees of freedom and Y' can be modeled as a sum of $\left(N_{\Omega_k} - 1\right)$ chi-square random variable with 2 degrees of freedom. Thus, the characteristic function of X and Y' can be represented respectively as [15]

$$\mathbf{\Phi}_X(\omega) = E\left\{\exp\left(j\omega X\right)\right\} = \frac{1}{1 - j\omega\lambda_{i,\max}^{k} P_{i,k}}, \tag{A.5}$$

$$\begin{aligned} \mathbf{\Phi}_{Y'}(\omega) &= E\left\{\exp\left(j\omega Y'\right)\right\} \\ &= E\left\{\exp\left(\sum_{u\in\Omega_k,\, u\neq i} j\omega\left|\mathbf{h}_{u,k}(t)\mathbf{w}_{u,k}^{\min}\right|^2 P_{u,k}\right)\right\} \\ &= \prod_{u\in\Omega_k,\, u\neq i}\frac{1}{1 - j\omega\lambda_{u,k}^{\min} P_{u,k}}. \end{aligned} \tag{A.6}$$

Although probability density function (pdf) of a random variable can be derived by its characteristic function, the pdf of Y' is not easily driven when $N_{\Omega_k} \geq 4$. In practice, however, most of interference may come from one or two neighboring cells in the cell boundary region. Thus, we only consider the case when $N_{\Omega_k} = 2$ or 3.

Let $Y \triangleq Y' + I'_k$ in (A.2). Since the pdf of $Z = X/Y$, where X and Y are independent random variables, is given by [15]

$$f_Z(z) = \int_{-\infty}^{\infty} |y|\, f_X(yz) f_Y(y) dy, \tag{A.7}$$

the pdf of $\gamma_{i,k}$ can be given by (A.8), at the bottom of the page, where α and β denote the index of non-primary cells.

Thus, the channel capacity $C\left(\Omega_k\right)$ of the proposed scheme in single user environments for a given active set Ω_k can be calculated by

$$C\left(\Omega_k\right) = E\left\{\log_2(1 + \gamma_{i,k})\right\}. \tag{A.9}$$

It cannot be represented in a closed form, but it can be represented by an upper bound given as

$$C\left(\Omega_k\right) \leq \log_2\left(1 + E\left\{\gamma_{i,k}\right\}\right) \triangleq C_{upper}\left(\Omega_k\right) \tag{A.10}$$

where $E\left\{\gamma_{i,k}\right\}$ is shown in (A.11), at the top of the next page and $Ei\left(z\right) = \int_z^{\infty} e^{-t}/t\,dt$. Note that when $N_{\Omega_k} = 1$, $E\left\{\gamma_{i,k}\right\}$ can easily be obtained by simply averaging $\gamma_{i,k}$.

$$f_{\gamma_{i,k}}(z) = \begin{cases} \dfrac{\left(I'_k + \dfrac{\lambda_{i,k}^{\max}P_{i,k}\lambda_{\alpha,k}^{\min}P_{\alpha,k}}{\lambda_{\alpha,k}^{\min}P_{\alpha,k}z + \lambda_{i,k}^{\max}P_{i,k}}\right)\exp\left(-\dfrac{I'_k}{\lambda_{i,k}^{\max}P_{i,k}}z\right)}{\lambda_{\alpha,k}^{\min}P_{\alpha,k}z + \lambda_{i,k}^{\max}P_{i,k}}, & \text{when } N_{\Omega_k} = 2 \\[2em] \dfrac{\lambda_{i,k}^{\max}P_{i,k}\left(I'_k + \dfrac{\lambda_{i,k}^{\max}P_{i,k}\lambda_{\alpha,k}^{\min}P_{\alpha,k}}{\lambda_{\alpha,k}^{\min}P_{\alpha,k}z + \lambda_{i,k}^{\max}P_{i,k}} + \dfrac{\lambda_{i,k}^{\max}P_{i,k}\lambda_{\beta,k}^{\min}P_{\beta,k}}{\lambda_{\beta,k}^{\min}P_{\beta,k}z + \lambda_{i,k}^{\max}P_{i,k}}\right)\exp\left(-\dfrac{I'_k}{\lambda_{i,k}^{\max}P_{i,k}}z\right)}{\left(\lambda_{\alpha,k}^{\min}P_{\alpha,k}z + \lambda_{i,k}^{\max}P_{i,k}\right)\left(\lambda_{\beta,k}^{\min}P_{\beta,k}z + \lambda_{i,k}^{\max}P_{i,k}\right)}, & \text{when } N_{\Omega_k} = 3. \end{cases} \tag{A.8}$$

$$E\left\{\gamma_{i,k}\right\}=\begin{cases}\dfrac{\lambda_{i,k}^{\max}P_{i,k}Ei\left(\dfrac{I'_k}{\lambda_{\alpha,k}^{\min}P_{\alpha,k}}\right)\exp\left(\dfrac{I'_k}{\lambda_{\alpha,k}^{\min}P_{\alpha,k}}\right)}{\lambda_{\alpha,k}^{\min}P_{\alpha,k}}, & \text{when } N_{\Omega_k}=2\\[2ex] \dfrac{\lambda_{i,k}^{\max}P_{i,k}\left(Ei\left(\dfrac{I'_k}{\lambda_{\alpha,k}^{\min}P_{\alpha,k}}\right)\exp\left(\dfrac{I'_k}{\lambda_{\alpha,k}^{\min}P_{\alpha,k}}\right)-Ei\left(\dfrac{I'_k}{\lambda_{\beta,k}^{\min}P_{\beta,k}}\right)\exp\left(\dfrac{I'_k}{\lambda_{\beta,k}^{\min}P_{\beta,k}}\right)\right)}{\lambda_{\alpha,k}^{\min}P_{\alpha,k}-\lambda_{\beta,k}^{\min}P_{\beta,k}}, & \text{when } N_{\Omega_k}=3\end{cases} \tag{A.11}$$

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